INSTITUTE FOR PHYSICAL AI @ BMI
The Charlot Lab & The Hiner Lab

Defensive Publication DP-2026-01
CC0 1.0 · public-domain dedication
6 July 2026

DEFENSIVE PUBLICATION · HEAT–CHARGE–INFORMATION

Joule-Priced Co-Optimization of a Shared Microfluidic Substrate for Physical AI

One embedded electrolyte network, optimized jointly as coolant, power medium, and iontronic computer. Pricing any current alone is dominated whenever the couplings below are nonzero.

David Jean Charlot, PhD · Michael Hiner

The Charlot Lab & The Hiner Lab, Institute for Physical AI @ BMI

Correspondence: contact@physicalai-bmi.org · physicalai-bmi.org · Interactive companion: physicalai-bmi.org/research/hiner-lab#topic-microfluidic

A single embedded electrolyte network is optimized jointly as coolant, localized power/regulation medium, and iontronic computational substrate. The three currents — heat, charge, information — share one flow and one dissipation budget. Under nonzero coupling (K1–K3, §4) the joint program below strictly dominates any pricing that optimizes cooling, power delivery, or computation in isolation. The disclosed object is the program itself: the objective, its ledger, the three couplings, and a determinism constraint that makes the readout bit-exact and safe. All constituent transfer functions are public physics; no term is enclosable.

1. Decision variables

Design (static) collects the channel diameters d_i and lengths ℓ_i, the manifold topology $\mathcal{T}$, electrode areas $A_{e,i}$, porosities φ_i, node geometries $(\mathbf{x}_k, a_k)$, and the per-node quantizer LSB δ_k:

$$\mathbf{g} = \{\, d_i,\ \ell_i,\ \mathcal{T},\ A_{e,i},\ \varphi_i,\ (\mathbf{x}_k, a_k),\ \delta_k \,\}.$$

Operating (control, time-varying) — the flows $Q_i(t)$, inlet concentration $\sigma_{in}(t)$ and temperature T_{in}, drive current $j(t)$, and node gates $\mathbf{v}_k(t)$ — with state fields $\mathbf{s}$:

$$\mathbf{u}(t) = \{\, Q_i(t),\ \sigma_{in}(t),\ T_{in},\ j(t),\ \mathbf{v}_k(t) \,\}, \qquad \mathbf{s}(\mathbf{x}, t) = (\mathbf{u}_f,\ p,\ T,\ \{c_s\},\ \phi).$$

2. Governing constraints (physics on domain Ω)

Momentum (Darcy–Brinkman; open channel and porous electrode):

$$\rho(\mathbf{u}_f \cdot \nabla)\mathbf{u}_f = -\nabla p + \mu \nabla^2 \mathbf{u}_f - \tfrac{\mu}{\kappa}\mathbf{u}_f, \qquad \nabla \cdot \mathbf{u}_f = 0.$$

Energy:

$$\rho c_p\big(\partial_t T + \mathbf{u}_f \cdot \nabla T\big) = \nabla\cdot(k\nabla T) + q_{gen} + q_{rxn}.$$

Species — Poisson–Nernst–Planck **with the Soret term**, per species s:

$$\partial_t c_s + \nabla\cdot \mathbf{J}_s = R_s, \qquad \mathbf{J}_s = \underbrace{c_s\mathbf{u}_f}_{\text{adv}} - \underbrace{D_s\nabla c_s}_{\text{diff}} - \underbrace{z_s\frac{D_sF}{RT}c_s\nabla\phi}_{\text{migr}} - \underbrace{D_s c_s S_{T,s}\nabla T}_{\text{Soret}}.$$

Poisson: $-\nabla\cdot(\varepsilon\nabla\phi) = F\sum_s z_s c_s$. Electrode kinetics (Butler–Volmer, temperature-dependent exchange current):

$$j = j_0(T)\left[e^{\alpha_a F\eta/RT} - e^{-\alpha_c F\eta/RT}\right], \qquad j_0(T) = j_0^{ref}\, e^{-\frac{E_a}{R}\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)}.$$

Iontronic readout at node k: $y_k = \mathcal{G}_k(\{c_s\}, T, \mathbf{v}_k)$ (local conductance/current functional, analog). **Quantized readout:** $\hat{y}_k = \mathsf{Q}(y_k)$ with Q a Q32.32 fixed-point quantizer of LSB δ. The committed observable is the word $\hat{y}_k$, not the analog y_k; sub-LSB drift is absorbed by Q. The trajectory digest is $H = \mathrm{SHA256}\big(\,\|_k\|_t\,\hat{y}_k(t)\,\big)$ over the canonical serialization of all node words.

3. Objective — net exergy destroyed per relevant outcome

$$\min_{\mathbf{g},\,\mathbf{u}(t)} \quad J_{\text{rel}} = \frac{\displaystyle\int_0^\tau \Big(P_{\text{comp}} + P_{\text{pump}} - P_{\text{fc}} - \eta_{\text{ex}}^{\text{self}}\dot{X}_{\text{th}}^{\text{self}} - \eta_{\text{ex}}^{\text{ext}}\dot{X}_{\text{th}}^{\text{ext}}\Big)\,dt}{\displaystyle\int_0^\tau \Lambda\big(\{y_k\}, t\big)\,dt}$$

TERM	DEFINITION	ROLE
$P_{\text{comp}} = \sum_k v_k i_k$	iontronic switching + CMOS	compute cost; floored by Landauer
$P_{\text{pump}} = \frac{1}{\eta_p}\sum_i Q_i\,\Delta p_i$	hydraulic work	**debit**
$P_{\text{fc}} = \int_{A_e} j\,(E_{eq} - \eta_{act} - \eta_{ohm} - \eta_{conc})\,dA$	flow-cell delivery	**charge credit**
$\dot{X}_{\text{th}}^{\text{self}}$	outlet availability recirculated to upstream electrode	**self-recovery credit** — closes in the network, drives K1
$\dot{X}_{\text{th}}^{\text{ext}}$	outlet availability exported to a *named* external sink	**export credit** — weaker; defaults to 0
$\Lambda \le N_{node}/\tau_{ion}(T, \mathbf{g})$	utility-weighted throughput	relevant outcomes / s

with $\dot{X}_{\text{th}}^{\text{self}} + \dot{X}_{\text{th}}^{\text{ext}} = \dot{m}\,c_p\left[(T_{out} - T_0) - T_0\ln\frac{T_{out}}{T_0}\right]$. Only the self term is defensible without a counterparty; a reader cannot collapse export into self. Set $\eta_{\text{ex}}^{\text{ext}} = 0$ for the standalone-die claim.

4. The three coupling terms

K1 — thermal → charge, positive (the exergy-recovery mechanism):

$$\frac{\partial P_{\text{fc}}}{\partial T} > 0 \quad \text{via} \quad \frac{\partial j_0}{\partial T} > 0,\ \ \frac{\partial E_{eq}}{\partial T}\ (\text{Nernst}).$$

Removed heat upgrades delivered charge; dissipation becomes a credit, not pure loss.

K2 — flow → cooling vs. power vs. pumping, adversarial:

$$\frac{\partial T_{\max}}{\partial Q} < 0, \qquad \frac{\partial \xi}{\partial Q} < 0, \qquad \frac{\partial P_{\text{pump}}}{\partial Q} > 0.$$

One scalar Q, three opposed partials (ξ = residence-limited utilization). A second adversarial scalar is the per-node quantizer LSB δ_k:

$$\frac{\partial\,(\text{margin}_5)}{\partial \delta_k} > 0, \qquad \frac{\partial \Lambda}{\partial \delta_k} < 0\ \ (\Lambda \propto \log_2(\text{range}/\delta_k)).$$

Constraint 5 lower-bounds it: $\delta_k \geq \delta_k^{\min}(\mathcal{F}) = 2\sup_{\mathcal{F}}|y_k - \bar{y}_k|$, and Λ wants it small, so the optimum pins $\delta_k = \delta_k^{\min}(\mathcal{F})$ — quantization rides the invariance floor. Any widening of the feasible envelope $\mathcal{F}$ for K2 cooling/power freedom inflates $\sup_{\mathcal{F}}|y_k - \bar{y}_k|$, raises $\delta_k^{\min}$, and pays in Λ. **Cooling headroom is charged, through invariance, against readout resolution** — not through the energy ledger.

K3 — thermal ↔ information, bidirectional (Soret):

$$\mathbf{J}_s^{\text{Soret}} = -D_s c_s S_{T,s}\nabla T\ \Rightarrow\ \nabla T \text{ writes } c_s \text{ writes } y_k; \qquad q_{gen} = P_{\text{comp}} \text{ feeds } T.$$

The thermal-management field is a computational input, and computation is a distributed heat source. ∇T is a **controlled design input**, not a nuisance to be uniformly suppressed. Two disjoint regimes are claimed: **(a) write mode** — ∇T shaped at node k to program c_s through the Soret flux (the cooling field doubles as the write bus); **(b) invariance mode** — ∇T held flat within tolerance so readout depends only on $\mathbf{v}_k$. Constraint 2 bounds $T_{\max}$ in both; it does not force $\nabla T \to 0$ globally. (Secondary: $\partial \tau_{ion}/\partial T < 0$ — warmth also accelerates Λ.)

5. Constraints

1. **Physics:** all §2 PDEs hold on Ω with boundary conditions.
2. **Reliability:** $T(\mathbf{x}, t) \leq T_{\max}\ \forall\, \mathbf{x}, t$.
3. **Hydraulic:** $\Delta p_i \leq \Delta p_{\max},\ Q_i \geq 0$.
4. **Electrochemical:** $\sigma \in [\sigma_{\min}, \sigma_{\max}],\ j \leq j_{\text{lim}}(Q, c)$ (mass-transport limit).
5. **Determinism / hardware-invariance (hard, non-tradeable):** bit-exact, hash-checkable, no tolerance band. $H(\mathbf{u}, \mathbf{g}) = H^{\text{ref}} \Leftrightarrow \hat{y}_k = \hat{y}_k^{\text{ref}}\ \forall\, k, t,\ \forall(\mathbf{g}, \mathbf{u}) \in \mathcal{F}$. The reference is a fixed-point word, so invariance is verified by SHA-256 equality of the trajectory digest, not an analog ϵ; δ must exceed the worst-case spread, $\sup_{\mathcal{F}}|y_k - \bar{y}_k| < \delta/2$. Operating point may change *speed and cost*, never the emitted word. This is the physical-AI-safety constraint; it does not trade against J_{rel}.
6. **Thermodynamic floor:** $J_{\text{rel}} \geq k_B T \ln 2$ per irreversible bit (feasibility bound).
7. **Manufacturability:** $\mathbf{g} \in \mathcal{G}_{\text{fab}}$ (open-PDK / soft-litho realizable).

6. Claim

The three currents share one network and one dissipation budget. Under $K1, K2, K3 \neq 0$, any pricing that optimizes cooling, power delivery, or computation independently is provably dominated by the joint program above. The disclosed object is the program itself — objective, ledger, K1–K3, and the determinism constraint — priced in exergy rather than energy. All constituent transfer functions ($j_0(T)$, $S_{T,s}$, $\tau_{ion}(T)$, $E_{eq}(T, \sigma)$) are public physics; no term is enclosable.

Selected prior art

1. R. van Erp, R. Soleimanzadeh, L. Nela, G. Kampitsis, E. Matioli. Co-designing electronics with microfluidics for more sustainable cooling. *Nature* 585, 2020. doi:10.1038/s41586-020-2666-1.
2. T. M. Kamsma, W. Q. Boon, T. ter Rele, C. Spitoni, R. van Roij. Iontronic Neuromorphic Signaling with Conical Microfluidic Memristors. *Phys. Rev. Lett.* 130, 268401, 2023. arXiv:2301.06158.
3. T. M. Kamsma et al. Brain-inspired computing with fluidic iontronic nanochannels. *PNAS* 121, e2320242121, 2024.
4. R. Landauer. Irreversibility and heat generation in the computing process. *IBM J. Res. Dev.*, 1961.

AI-use disclosure. Preparation of this defensive publication used a large language model (Claude, Anthropic) for drafting, editing, typesetting the equations, and the interactive companion. The authors reviewed the content, verified the cited prior art, and are solely responsible for the work. Consistent with ICMJE, COPE, and IEEE guidance, the model is a tool and is not credited as an author.

Institute for Physical AI @ BMI · The Charlot Lab & The Hiner Lab
503 McKeever Rd, Arcola, TX 77583, USA
physicalai-bmi.org · contact@physicalai-bmi.org

Defensive Publication DP-2026-01