INSTITUTE FOR PHYSICAL AI @ BMI
The Charlot Lab

Technical Report TR-2026-20
Survey / Review · Preprint v1
15 July 2026

EXPLORING INTELLIGENCE

Impedance Matching as a Unifying Principle for Physical and Embodied AI: A Survey

One boundary condition, carried from a 1939 transmission-line calculator to the free-energy principle — and an account of where it stops being one.

David Jean Charlot, PhD

Dean of Physical AI · The Charlot Lab, Institute for Physical AI @ BMI

Correspondence: contact@physicalai-bmi.org · physicalai-bmi.org · The Charlot Lab: labs.physicalai-bmi.org/charlot
Interactive companion: physicalai-bmi.org/education/exploring-intelligence · Instruments: physicalai-bmi.org/assets/sims/smith-chart, /assets/sims/contact-impedance

Abstract. A wave meets a boundary; whatever fails to match is reflected. This report surveys the extent to which that single statement organizes problems across the disciplines that physical and embodied AI draws on. It reviews five bodies of published work — the reflection coefficient and its conformal map onto the unit disk, Shannon capacity and the matched filter, the Bode–Fano broadband matching limit, Landauer's bound on erasure, and the Colgate–Hogan passivity condition for contact — and identifies which relations among them are mathematical identities and which are only analogies. The central identity the survey emphasizes is not new: the scattering operator that certifies a robot's interaction port as passive, $S = (Z-1)(Z+1)^{-1}$, is the same Möbius transformation that maps impedance onto the reflection coefficient, so the classical condition for safe contact with an arbitrary passive environment is the statement that the port's locus remains within the unit disk. Section 2 states the review method and the limits of the claim. Sections 3 through 9 survey the domains. Section 10 reviews learned interfaces. Section 11 is a deliberate account of where the unification is exact, where it is a useful analogy, and where it is rhetoric; it argues that the free-energy reading, the most quoted step, is also the weakest. Section 12 records open questions, including whether any Bode–Fano-like conservation law governs policy robustness across an environment-parameter band. The report is a survey and a research position; it reports no new experimental measurements.

1. Introduction

In 1928 an engineer at Bell Laboratories was asked to deliver radio power down roughly two kilometres of transmission line into a directional antenna array, and found that part of the signal kept returning to him. The reflected wave met the forward wave and produced a standing pattern, with voltage peaks capable of destroying the line. The cause was a mismatch of impedance at a boundary. The instrument Phillip Smith eventually published to reason about it [1] is a conformal map that turns an unbounded impedance plane into a finite disk, and it is still in use.

The observation this report examines is that the same boundary condition, with the same algebra, recurs in several fields that physical and embodied AI now spans. A channel that is not matched to its receiver loses information. A load that is reactive cannot be matched across a band without paying elsewhere. A bit that is erased must dissipate heat. A robot whose interaction port is not passive will chatter against a wall. In each case a quantity crosses a boundary and a portion of it fails to cross.

The purpose here is not to assert that these are the same phenomenon. Some of them are related by identity, some by structural analogy, and at least one by nothing stronger than a shared vocabulary. Distinguishing among those three is the contribution this report intends. The survey is written alongside a course and two interactive instruments, in which every quantitative statement below can be recomputed by the reader; the report itself reports no new measurements.

2. Scope and method

This is a review of published prior art together with a research position. Its sources are the primary papers of each field, cited in place. It contains no experimental results, no proprietary methods, and no claim of priority. Where a relation between two fields is a theorem, this report says so and cites the theorem. Where the relation is an analogy that has been productive but is not proved, this report labels it an analogy. Where a relation is asserted in informal literature but this author could not substantiate it, Section 12 records it as open.

Three exclusions are worth stating. First, the report treats one-port, linear, time-invariant relations; most of the identities below fail or require care outside that class, and Section 11 says where. Second, it does not survey the enormous impedance-matching literature of microwave engineering for its own sake, only insofar as it bears on the through-line. Third, it takes no position on the empirical adequacy of the free-energy principle as a theory of brain function, a question outside its competence; it examines only whether the matching reading of it is mathematically meaningful.

3. The reflection coefficient and the disk

For a one-port of impedance Z presented to a line of real characteristic impedance Z_0, the reflection coefficient is $\Gamma = (Z - Z_0)/(Z + Z_0)$. This is a Möbius transformation, hence conformal, and it carries the closed right half-plane of the impedance variable onto the closed unit disk. The correspondence is exact and elementary: $\mathrm{Re}\, Z \geq 0$ if and only if $|\Gamma| \leq 1$. Passive impedances live in the disk; the boundary is the lossless case; the exterior is active.

Two properties give the map its durability. It compactifies: the point at infinity — an open circuit — becomes an ordinary point on the rim, so an unbounded plane is drawn on a page. And it is conformal, so the constant-resistance and constant-reactance lines of the impedance plane arrive as the orthogonal circle families of the chart. Smith's construction [1] is a nomogram for exactly this map.

Two generalizations matter later. When the reference impedance is complex — a source with reactance — the naive expression no longer vanishes at the condition of maximum power transfer. Kurokawa's power-wave formulation [2] replaces the numerator with $Z - Z_0^*$, restoring the property that a match is $\Gamma = 0$; the conjugate is what makes the map still mean "nothing comes back". And the disk itself has been extended: work by Müller and colleagues has restored the projective object the chart is a shadow of, rendering the sphere so that active loads with $|\Gamma| > 1$ acquire a home, and has developed hyperbolic and generalized forms [3]. The recurring lesson from that program is that the disk is not a finished graphic but a geometry practitioners keep extending.

4. The information domain: capacity and the matched filter

Shannon bounded what any channel can carry [4]: with bandwidth B and signal-to-noise ratio, capacity is $C = B \log_2(1 + \text{SNR})$. Bandwidth and signal-to-noise are the only currencies, and a receiver's job is to reach the ceiling that they set.

The instrument for that is the matched filter [5]: among all linear filters of fixed energy, output signal-to-noise ratio is maximized by the one whose impulse response is the time-reverse of the expected signal, achieving $2E/N_0$ in white noise. The proof is the Cauchy–Schwarz inequality: an inner product is largest when its arguments are parallel.

It is common, and defensible, to describe this as conjugate matching moved into the information domain. The defensible part is precise: both are optimizations of a bilinear quantity subject to a norm constraint, and both are solved by alignment with a conjugate. The part that should not be overstated is that the matched filter is not derived from network theory and does not require it; the shared structure is the variational statement, not the physics. This is the first place in the survey where a real correspondence and a rhetorical one sit close together.

5. The bandwidth budget: Bode–Fano

Matching at a point is easy; matching across a band is bounded. For a reactive load, Fano's analysis [6], building on Bode [7], establishes that the integral of $\ln(1/|\Gamma|)$ over frequency cannot exceed a constant fixed by the load — for a parallel RC, π/RC. No network, of any order, evades it. Matching therefore becomes accounting: the budget is set by the load, and depth of match in one band is purchased with reflection in another.

The result has a well-known sibling in feedback control, Bode's sensitivity integral, in which suppressing sensitivity over one frequency range necessarily raises it elsewhere; the informal name is the waterbed effect. The two are not the same theorem, and this report does not claim they are; they are two instances of a conserved integral arising from analyticity and a Poisson-type relation on the half-plane. The honest statement is that analyticity plus passivity yields conservation laws of this shape, and that both results are members of that family.

6. The thermodynamic domain: the price of a bit

Landauer argued that logical irreversibility carries a thermodynamic cost: erasing one bit at temperature T dissipates at least $k_B T \ln 2$ [8]. Bennett later showed that computation as such need not be irreversible, so the bound applies to erasure rather than to logic in general [9]. The bound was measured directly by Bérut and colleagues in a colloidal double-well experiment [10], and approached only in the quasi-static limit; finite-time erasure dissipates more.

This is where the informational spine of the survey meets a physical one, and the meeting is a genuine theorem rather than an analogy. It is also where the matching language becomes strained, and the strain is instructive. One can describe erasure as an unmatched boundary between a logical state space and a thermal bath, and the description is not false, but it does no work: the bound follows from the entropy accounting, not from any impedance. This report cites Landauer because a survey of what information costs the physical world cannot omit it, not because the reflection picture explains it.

7. The mechanical domain: impedance, contact, and passivity

Under the standard analogy — force as voltage, velocity as current — mechanical impedance is $Z = F/v$, dampers are resistors, masses are inductive, springs are capacitive, and a gearbox of ratio n is a transformer scaling impedance by n^2. The analogy is not loose: it is an isomorphism of the constitutive equations, which is why the maximum-power-transfer theorem arrives in servo engineering as inertia matching, the rule that acceleration is maximized when the load inertia reflected through the gearbox equals the rotor's own.

Hogan's impedance control [11] made the port variable the object of control rather than position or force alone. Colgate and Hogan then established the result this report treats as the anchor of the mechanical case [12]: a manipulator is stable in contact with *every* passive environment if and only if its own interaction port is passive, that is $\mathrm{Re}\, Z(j\omega) \geq 0$ at all frequencies. Because the scattering operator $S = (Z - 1)(Z + 1)^{-1}$ is bounded by unity exactly when the impedance is positive real, the condition is that the port's locus stays inside the unit disk. That is the same Möbius map as Section 3, and the correspondence is an identity, not a metaphor. Chattering contact is not a failure to match well; it is a departure from the disk.

Digital implementation is what pushes ports out of the disk. A sampled controller holds its commanded force for a period T, and a held spring is a spring whose force lags the motion; to first order that lag acts as a negative damper. Colgate and Schenkel analysed the passivity of this class of sampled-data systems and produced the practical rule that physical damping must exceed a term proportional to KT [13]. The conclusion practitioners draw is that sample rate buys renderable stiffness. Colgate and Brown had already named the quantity that this bounds: the Z-width of a haptic display, the region of impedance it can present while remaining passive [14] — in the language of this report, its matchable region.

One caution belongs here, because it is easy to state the rule too confidently. The exact form of the sampled-data condition depends on how velocity is obtained. With a backward difference on a quantized encoder, virtual damping is itself destabilizing and appears on the wrong side of the inequality; with an ideal velocity subject only to the hold, it is stabilizing. Both are reported in the literature and both are correct for their own discretization. A reader should determine which case applies to their hardware rather than quote a single inequality, and the accompanying instrument is built so that the boundary can be measured rather than assumed.

8. Teleoperation and wave variables

A communication delay in a force-reflecting loop destabilizes contact. Anderson and Spong diagnosed this in scattering terms and proposed a passivating architecture [15]; Niemeyer and Slotine recast the transmitted signals as wave variables against a wave impedance b [16], so that the channel behaves as a matched transmission line and delay costs phase rather than stability. The vocabulary here is not borrowed: the analysis is scattering theory applied to a control loop, and the terminating impedance is chosen to match, exactly as a line is terminated.

This is the cleanest transplant in the survey, and it also carries a caution the survey should not omit: the passivity of the wave transform is a continuous-time result, and its discrete implementation raises energy-leak questions of the same family as Section 7's hold. Practitioners report needing care there. This report does not resolve that; it flags it.

9. The body as a matching layer

Not all of an animal's control is computed in its nervous system. A tendon that stores energy, a fingertip that conforms, a leg that rebounds: each performs work that would otherwise fall to a controller, at the speed of physics and without sensing. Pfeifer and Bongard's account of embodiment [17] and the morphological-computation literature that followed [18] make the case that morphology is a computational resource.

Read through this report's lens, compliant structures are matching layers: they reshape the impedance a robot presents at its port so that contact proceeds quietly and so that geometry, rather than estimation, resolves small misalignments. Series elastic actuators, soft fingertips, and compliant wrists all admit that reading. The reading is a reading, not a theorem — morphological computation is a broader claim than impedance shaping, and not every instance of it is an impedance argument.

10. Learned interfaces

The chart has moved down the stack it helped build. At millimetre wavelengths on-chip interconnect is transmission line, so matching networks are fabricated on the die and made tunable; plasma etch and deposition chambers are driven through radio-frequency matching networks that retune as the plasma's impedance moves during a pulse; superconducting qubit readout is gated by the match of the amplifier chain. There is a certain closure in noting that the transistors executing this sentence were etched behind a live matching loop.

More relevant to this report is that the geometry has become a representation to compute in rather than only to read. Work on neural network models operating in chart coordinates, and on reinforcement-learning and graph-neural-network methods for radio-frequency circuit design, treats the chart's structure as an inductive bias. On the embodied side, manipulation policies increasingly emit stiffness and damping rather than positions, which places a policy's action in the same space this report has been describing; a contact task is then a trajectory across that space. Energy-tank and passivity-filter methods meter the energy crossing the contact port and clamp the commanded impedance when a budget is exhausted, which is a runtime constraint on the port rather than a verification of the network. That direction — leaving the policy unverified and instead bounding the boundary it acts through — is, in this author's view, the most underweighted idea in physical-AI safety, and it is offered here as a position rather than as a result.

11. Discussion: what is identity, what is analogy, what is rhetoric

A unifying principle that explains everything explains nothing, so this section states the grades explicitly.

Identity. Sections 3 and 7 are joined by a theorem. The positive-real condition and the unit-disk condition are the same statement under a Möbius map, and the classical passivity result for contact is therefore literally a statement about a Smith chart in mechanical dress. The electrical–mechanical isomorphism is an isomorphism of constitutive equations, and inertia matching is the maximum-power-transfer theorem. Section 8 is scattering theory in a control loop. Nothing here is metaphor.

Structural analogy. The matched filter and conjugate matching share a variational skeleton but not a derivation. Bode–Fano and Bode's sensitivity integral belong to a family of conservation laws that follow from analyticity, but are not one theorem. These are productive analogies that have repeatedly suggested correct moves; they are not licences to transfer results.

Rhetoric. The free-energy reading is the weakest link, and it is the one most likely to be quoted. Under a linear-Gaussian model, minimizing variational free energy with respect to a belief recovers the Bayesian posterior, and minimizing the same quantity by acting drives the world toward the model's expectation — a clean statement of perception and action as two descents of one objective [19]. To call the residual surprise a "reflected wave" is evocative and, this author judges, currently no more than that. There is no impedance in the derivation, no boundary in the network sense, and no conservation law being invoked. It may be that a scattering formulation of an agent–environment boundary can be constructed and made to do work; until it is, the honest description of this step is that the vocabulary rhymes.

The limits of the whole picture are worth stating plainly. Every identity in Section 3 and Section 7 is linear and time-invariant, and one-port. Contact is neither: it is unilateral, intermittent, and nonlinear in the extreme. Passivity is a sufficient condition for stability against passive environments and is conservative — real robots exploit environments that are not adversarial, and passivity forbids useful behaviours. And the framing is silent about what to match *to*: an impedance argument says how to deliver power to a load, never which load is worth powering. A theory of intelligence that cannot say what the task is has not yet said much.

12. Open questions

Three, stated as questions rather than claims. First: is there a Bode–Fano-like conservation law for policy robustness? The empirical pattern in domain randomization — widening the randomization band depresses peak performance — has the shape of a budget, and the informal literature occasionally asserts the connection. This author could not substantiate a proved integral of that form and records it as open; establishing or refuting one would put a number on the sim-to-real tax. Second: can the discrete-time passivity of wave-variable channels be given the same clean statement that the sampled-data hold has, so that a delayed teleoperation link comes with a rate condition rather than a tuning practice? Third: does a scattering formulation of an agent–environment boundary exist that reduces to the free-energy account and predicts something the free-energy account does not? A negative answer to the third would be worth as much as a positive one, and would properly retire the metaphor.

13. Conclusion

A wave meets a boundary, and whatever does not match comes back. Across the fields surveyed here that sentence is, in turn, a theorem, a productive analogy, and a figure of speech, and this report's position is that saying which is which is more useful than asserting the unification. The strongest version of the claim is narrow and secure: the condition for a robot to touch an unknown passive world safely is that its port impedance stay inside the unit disk, and that disk is the one Phillip Smith drew for antennas. The broadest version — that becoming intelligent is reshaping an interface until the reflection vanishes — is at present a research programme and a teaching device rather than a result. Both are offered here, labelled.

References

1. P. H. Smith. Transmission Line Calculator. *Electronics*, 12(1):29–31, Jan. 1939; and An Improved Transmission Line Calculator, *Electronics*, 17(1):130–133, 318–325, Jan. 1944. Pre-DOI; scans: 1939, 1944.
2. K. Kurokawa. Power Waves and the Scattering Matrix. *IEEE Transactions on Microwave Theory and Techniques*, 13(2):194–202, Mar. 1965. doi:10.1109/TMTT.1965.1125964.
3. A. A. Muller, P. Soto, D. Dascalu, D. Neculoiu, V. E. Boria. A 3-D Smith Chart Based on the Riemann Sphere for Active and Passive Microwave Circuits. *IEEE Microwave and Wireless Components Letters*, 21(6):286–288, Jun. 2011. doi:10.1109/LMWC.2011.2132697. Hyperbolic and generalized forms: A. A. Müller, E. Sanabria-Codesal, A. Moldoveanu, V. Asavei, D. Dascălu. Two Compact Smith Charts: The 3D Smith Chart and a Hyperbolic Disc Model of the Generalized Infinite Smith Chart. *Romanian Journal of Information Science and Technology*, 19(1–2):166–174, 2016. Review: M. J. Pérez-Peñalver, E. Sanabria-Codesal, A. A. Müller, et al. A Review and Mathematical Treatment of Infinity on the Smith Chart, 3D Smith Chart and Hyperbolic Smith Chart. *Symmetry*, 10(10):458, 2018. doi:10.3390/sym10100458. Retrospective: A. A. Muller et al. The 3D Smith Chart: From Theory to Experimental Reality. *IEEE Microwave Magazine*, 21(11):22–35, Nov. 2020. doi:10.1109/MMM.2020.3014984.
4. C. E. Shannon. A Mathematical Theory of Communication. *Bell System Technical Journal*, 27(3):379–423, Jul. 1948, doi:10.1002/j.1538-7305.1948.tb01338.x; Part II, 27(4):623–656, Oct. 1948, doi:10.1002/j.1538-7305.1948.tb00917.x.
5. G. L. Turin. An Introduction to Matched Filters. *IRE Transactions on Information Theory*, IT-6(3):311–329, Jun. 1960. doi:10.1109/TIT.1960.1057571.
6. R. M. Fano. Theoretical Limitations on the Broadband Matching of Arbitrary Impedances. *Journal of the Franklin Institute*, 249(1):57–83, Jan. 1950, doi:10.1016/0016-0032(50)90006-8; Part II, 249(2):139–154, Feb. 1950, doi:10.1016/S0016-0032(50)91101-X.
7. H. W. Bode. *Network Analysis and Feedback Amplifier Design*. D. Van Nostrand, New York, 1945, 551 pp. (Bell Telephone Laboratories Series).
8. R. Landauer. Irreversibility and Heat Generation in the Computing Process. *IBM Journal of Research and Development*, 5(3):183–191, Jul. 1961. doi:10.1147/rd.53.0183.

9. C. H. Bennett. Logical Reversibility of Computation. *IBM Journal of Research and Development*, 17(6):525–532, Nov. 1973. doi:10.1147/rd.176.0525.
10. A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, E. Lutz. Experimental Verification of Landauer's Principle Linking Information and Thermodynamics. *Nature*, 483(7388):187–189, Mar. 2012. doi:10.1038/nature10872.
11. N. Hogan. Impedance Control: An Approach to Manipulation. *ASME Journal of Dynamic Systems, Measurement, and Control*, 107(1), Mar. 1985 — Part I: Theory, 1–7, doi:10.1115/1.3140702; Part II: Implementation, 8–16, doi:10.1115/1.3140713; Part III: Applications, 17–24, doi:10.1115/1.3140701.
12. J. E. Colgate and N. Hogan. Robust Control of Dynamically Interacting Systems. *International Journal of Control*, 48(1):65–88, Jul. 1988. doi:10.1080/00207178808906161.
13. J. E. Colgate and G. G. Schenkel. Passivity of a Class of Sampled-Data Systems: Application to Haptic Interfaces. *Journal of Robotic Systems*, 14(1):37–47, Jan. 1997. doi:10.1002/(SICI)1097-4563(199701)14:1<37::AID-ROB4>3.0.CO;2-V.
14. J. E. Colgate and J. M. Brown. Factors Affecting the Z-Width of a Haptic Display. In *Proc. 1994 IEEE International Conference on Robotics and Automation (ICRA)*, San Diego, CA, pp. 3205–3210, 1994. doi:10.1109/ROBOT.1994.351077.
15. R. J. Anderson and M. W. Spong. Bilateral Control of Teleoperators with Time Delay. *IEEE Transactions on Automatic Control*, 34(5):494–501, May 1989. doi:10.1109/9.24201.
16. G. Niemeyer and J.-J. E. Slotine. Stable Adaptive Teleoperation. *IEEE Journal of Oceanic Engineering*, 16(1):152–162, Jan. 1991. doi:10.1109/48.64895.
17. R. Pfeifer and J. Bongard. *How the Body Shapes the Way We Think: A New View of Intelligence*. MIT Press, Cambridge, MA, 2006. doi:10.7551/mitpress/3585.001.0001.
18. H. Hauser, A. J. Ijspeert, R. M. Füchslin, R. Pfeifer, W. Maass. Towards a Theoretical Foundation for Morphological Computation with Compliant Bodies. *Biological Cybernetics*, 105(5–6):355–370, Dec. 2011. doi:10.1007/s00422-012-0471-0. Companion: The Role of Feedback in Morphological Computation with Compliant Bodies. *Biological Cybernetics*, 106(10):595–613, 2012. doi:10.1007/s00422-012-0516-4.
19. K. Friston. The Free-Energy Principle: A Unified Brain Theory? *Nature Reviews Neuroscience*, 11(2):127–138, Feb. 2010. doi:10.1038/nrn2787.

AI-use disclosure. Preparation of this report used a large language model (Claude, Anthropic) for drafting and editing text, organizing the reviewed literature, and preparing the interactive companion. Cited references were checked to resolve to their sources. The author reviewed the content and is solely responsible for it. Consistent with ICMJE, COPE, and IEEE guidance, the model is a tool and is not credited as an author.

Institute for Physical AI @ BMI · The Charlot Lab
503 McKeever Rd, Arcola, TX 77583, USA
physicalai-bmi.org · contact@physicalai-bmi.org

Technical Report TR-2026-20 · Preprint v1